# The Application of Item Response Theory for Development of a Students' Attitude Scale Toward Mathematics 

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#### Abstract

Mathematics Education Study Program students should have a tendency to accept mathematics because it will support the achievement of mathematical competence in the fields of work, knowledge, and management. This study aims to construct, validate, and analyze the characteristics of attitude scale items, and determine students' attitudes towards mathematics. Aspects of student attitudes towards mathematics, were taken as namely: intrinsic motivation, enjoyment, anxiety, self-confidence, and value. The results of factor analysis show good model fit with the items measuring unidimensionality. Analysis of item characteristics was done using polytomous item response theory with a Partial Credit Model (PCM). The difficulty level of grains is at intervals of $-2.52 \leq \mathrm{d} \leq 2.58$. Students' attitudes towards mathematics are in the interval $-0.67 \leq \theta \leq 2.36$.


Key words: student attitudes towards mathematics, instrument construction, Item Response Theory, polytomous items, Partial Credit Model

## Introduction

Attitude is a concept associated with affective domains such as beliefs, emotions, values (McLeod, 1992; Zan, Brown, Evans \& Hannula, 2006; Goldin, Rösken, \& Törner, 2009). Attitudes can be described as tendencies that are associated with individuals and regularly form thoughts, feelings, and behaviors that are related to psychology (Dursun, 2015). Some researchers such as McLeod (1992) consider attitude to be a series of emotional reactions, while other researchers such as Di Martino \& Zan (2010) argue that several components interact with each other in attitude forming.

In mathematics education, early studies of attitudes began to emerge in the mid- $20^{\text {th }}$ century (Dutton, 1951). The assumption is that it is not only cognitive factors that play a role in mathematics learning. To this end, students of the Mathematics Education Study Program as prospective mathematics educators who are outstanding, creative, superior, professional, and globally competitive, must also pay attention to non-cognitive factors that will maximize their mathematical competence. Zan, Brown, Evans \& Hannula (2006) suggested that the most important research about mathematical attitudes is on understanding the mutual relationship between attitude and achievement. Several other studies show that there is a strong relationship between various domains of attitude (enjoyment of mathematics, motivation to carry out mathematical activities, confidence in mathematics, and perception of mathematical values) and mathematical achievement (Anttonen, 1969; Atkinson \& Raynor, 1974; Minato, 1983; Minato \& Yanase, 1984; Foire, 1999; Bouchey \& Harter, 2005; Samuelsson \& Granstrom, 2007).

Generally, students of the Mathematics Education Study Program have a positive tendency towards mathematics at the beginning when they choose this study program. High school graduates who choose to continue their studies in the Mathematics Education Study Program certainly already have a sense of interest in mathematics compared to other subjects. But in the course of being a student, these tendencies can change. Changes in attitude can occur at any time because of the functional value of attitude, and the process that changes this has become a major focus in social psychology (Petty, Wheeler, \& Tormala, 2003). It is hoped that students' attitudes towards mathematics will not turn negative. A negative attitude is considered one of the main factors that influence poor achievement in mathematics (Schoenfeld, 1989; Zan, Brown, Evans \& Hannula, 2006). Therefore, measuring the attitudes of students of the Mathematics Education Study Program towards mathematics is considered necessary so that students can continue to excel in mathematics.

## Research Problem

The relationship between attitude and achievement has been widely studied using instruments that measure attitudes towards mathematics and student achievement. However, the basis for concluding the relationship between attitudes and achievements is not yet strong, and the instruments used for this goal often do not go through the process required to develop correct instruments (Ma \& Kishor, 1997; McLeod, 1992).

## Research Focus

The subjects of this study were students of the Mathematics Education Study Program who had different characteristics from elementary and middle school students. Therefore, it is necessary to research the development of instruments with a scale of students' attitudes towards mathematics. The purpose of this study is to: 1 ) construct an attitude scale; 2) validate the attitude scale; 3 ) analyze the characteristics of the attitude scale items; and 4) discover students' attitudes towards mathematics.

## Methodology of Research

## Research Sample

The subjects of this study were 227 undergraduate Mathematics Education Study Program students of whom 127 (55.95\%) came from public universities and $100(44.05 \%)$ came from private universities. The sample consisted of 119 ( $52.42 \%$ ) male students and 108 ( $47.58 \%$ ) female students. There are 72 ( $31.72 \%$ ) $1^{\text {st }}$ semester students, 61 (26.87\%) 3rd semester students, 68 (29.96\%) $5^{\text {th }}$ semester students, $7(3.08 \%) 7^{\text {th }}$ semester students, with 19 students ( $8.37 \%$ ) people from higher than $8^{\text {th }}$ semester.

## Research Instrument Development

The scale of students' attitudes towards mathematics consists of 5 aspects, which are: intrinsic motivation, enjoyment, anxiety, self-confidence, and value. Each aspect was measured by 3 or 4 indicators. Indicators of intrinsic motivation are interest, desire in career choices, and enthusiasm to pursue mathematics beyond the mandatory level. The indicators of enjoyment are enjoying mathematics, participating in mathematical discussions, challenge in solving new problems, and happy feelings in math class. Indicators of anxiety are feelings of fear, nervousness, confusion, and feelings of tension. Self-confidence indicators are that the student
can perform well in mathematics, can learn mathematics easily, and can be a good problem solver. Value indicators are the usefulness of mathematics, the relevance of mathematics, and the value of mathematics in life.

The attitude scale was validated by 8 validators consisting of mathematics education lecturers, mathematics teachers, psychology lecturers, and linguists. The validator chose answers by paying attention to the appropriateness of aspects, indicators, and statements. All items are said to be valid because there was a coefficient value of Aiken's $\mathrm{V} \geq 0.75$ with a validator of 8 people and a choice of 4 answers (Aiken, 1985).

Instrument reliability related to measurement errors is indicated by Cronbach's Alpha coefficient of 0.816 (high reliability). High reliability indicates that there is a small error in obtaining measurement results. Item reliability is indicated by Corrected Item-Total Correlation. Corrected Item-Total Correlation in each item is more than 0.3 , from which it can be concluded that each item statement is reliable.

## Results of Research

In analyzing data using item response theory, the first thing to do is to test the dimensions of the empirical data. The testing process is carried out by exploratory factor analysis using the principal component method. The output shows that the Kaiser-Meyer-Olkin Measure of Sampling Adequacy $=0.790$ shows $\mathrm{KMO}>0.5$, which means that this study has sufficient data. Also, the significance of Bartlett's test shows that the correlation matrix is not an identity matrix, so the data form a correlation matrix with a close relationship between variables. Then based on anti-images correlation, the Measures of Adequate Sampling (MSA) was $>0.5$ so all data are eligible for factor analysis.

The number of factors can be determined by selecting factors that have an Eigenvalue greater than 1. Based on the Eigenvalue, 5 factors were formed. This can also be seen in the scree plot to determine the exact number of components, presented in Figure 1.

To facilitate interpretation, a rotation is carried out to obtain a simpler loading structure. The Varimax method was chosen to get a loading structure that has a strong relationship with only one factor. The results of the factor rotation are presented in Table 1. The results of the factor rotation indicate that each item measures 1 dimension so that subsequent data analysis uses unidimensional item response theory.

Scree Plot


Figure 1. Scree plot of the main component analysis

Table 1. Rotated component matrix

|  | Component |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  | 1 | 2 | 3 | 4 | 5 |  |
| A1 |  |  |  | 0.742 |  |  |
| A2 |  |  |  | 0.801 |  |  |
| A3 |  |  |  | 0.728 |  |  |
| B1 | 0.825 |  |  |  |  |  |
| B2 | 0.788 |  |  |  |  |  |
| B3 | 0.682 |  |  |  |  |  |
| B4 | 0.774 |  |  |  |  |  |
| C1 |  | 0.742 |  |  |  |  |
| C2 |  | 0.686 |  |  |  |  |
| C3 |  | 0.764 |  |  |  |  |
| C4 |  | 0.715 |  |  | 0.731 |  |
| D1 |  |  |  |  | 0.586 |  |
| D2 |  |  |  |  | 0.815 |  |
| D3 |  |  | 0.822 |  |  |  |
| E1 |  |  | 0.864 |  |  |  |
| E2 |  |  |  |  |  |  |
| E3 |  |  |  | 0.757 |  |  |

The validity of the construct theory of attitudes towards mathematics is evidenced by confirmatory factor analysis. The t -value output is presented in Figure 2.


Figure 2. T-value of the construct theory of attitude towards mathematics

Figure 2 shows all the paths are significant, but the p -value $=0.008$ for the model compatibility test is not significant. For this reason, it is necessary to pay attention to modification indices to find suggestions for improving the model. After modification, the standardized solution is presented in Figure 3.

To find out the characteristics of polytomous items, researchers used a Partial Credit Model (PCM) with the help of R software. The first step is to test the suitability of the model if the data analysis can use PCM. The results of the model compatibility test are presented in Table 2.


Figure 3. The standardized solution of the construct theory of attitude to mathematics after modification

Table 2. Model Match Test Output

| Item | Chisq | df | p-value | Outfit MSQ | Infit MSQ | Outfit t | Infit t |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1 | 231.569 | 226 | 0.385 | 1.02 | 1.013 | 0.25 | 0.18 |
| A2 | 225.346 | 226 | 0.500 | 0.993 | 0.995 | -0.05 | -0.03 |
| A3 | 201.128 | 226 | 0.882 | 0.886 | 0.87 | -1.24 | -1.46 |
| B1 | 217.182 | 226 | 0.651 | 0.957 | 0.982 | -0.45 | -0.17 |
| B2 | 194.622 | 226 | 0.936 | 0.857 | 0.878 | -1.66 | -1.43 |
| B3 | 231.064 | 226 | 0.394 | 1.018 | 1.034 | 0.23 | 0.4 |
| B4 | 199.263 | 226 | 0.900 | 0.878 | 0.888 | -1.51 | -1.37 |
| C1 | 232.509 | 226 | 0.369 | 1.024 | 1.044 | 0.3 | 0.53 |
| C2 | 221.735 | 226 | 0.568 | 0.977 | 1.013 | -0.22 | 0.17 |
| C3 | 183.638 | 226 | 0.982 | 0.809 | 0.818 | -2.41 | -2.39 |
| C4 | 185.679 | 226 | 0.977 | 0.818 | 0.828 | -2.17 | -2.06 |
| D1 | 204.705 | 226 | 0.842 | 0.902 | 0.91 | -1.19 | -1.11 |
| D2 | 231.335 | 226 | 0.390 | 1.019 | 1.001 | 0.22 | 0.04 |
| D3 | 256.819 | 226 | 0.078 | 1.131 | 1.071 | 1.31 | 0.76 |
| E1 | 207.82 | 226 | 0.802 | 0.916 | 0.941 | -0.91 | -0.64 |
| E2 | 214.295 | 226 | 0.702 | 0.944 | 0.974 | -0.64 | -0.28 |
| E3 | 220.587 | 226 | 0.589 | 0.972 | 0.993 | -0.26 | -0.04 |

The above output shows that there was a p-value> 0.05 for each item, which means that all items are fit to be analyzed using PCM which is a 1 PL polytomous model that considers the power of different items constant. Also, it is necessary to see the person fit to determine the characteristics of respondents who are suitable for this instrument to be used on. Of 227 respondents, there were 19 people (8.37\%) who did not fit the instrument construct.

The threshold is the intersection for each category that shows the minimum ability to be able to reach certain points. As there are 5 answer choices on the attitude scale, then there will be a maximum of 4 thresholds. The threshold for each item is presented in Table 3.

Table 3. The threshold for each item

| Item | Location | Threshold 1 | Threshold 2 | Threshold 3 | Threshold 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A1 | 0.326 | -0.358 | -0.336 | 0.687 | 1.309 |
| A2 | 0.352 | -0.604 | -0.256 | 0.747 | 1.520 |
| A3 | 0.375 | -0.691 | -0.028 | 0.478 | 1.743 |
| B1 | 0.448 | -0.054 | -0.717 | 0.893 | 1.671 |
| B2 | 0.305 | -1.193 | 0.222 | 0.509 | 1.681 |
| B3 | 0.445 | 0.047 | -0.701 | 0.956 | 1.477 |
| B4 | -0.010 | -1.850 | -0.669 | 0.855 | 1.624 |
| C1 | 0.287 | -0.707 | -0.332 | 0.807 | 1.379 |
| C2 | 0.401 | -0.417 | -0.481 | 0.600 | 1.901 |
| C3 | 0.635 | 0.044 | 0.552 | 1.308 | NA |
| C4 | 0.230 | -0.877 | -0.297 | 0.723 | 1.369 |
| D1 | 0.859 | 0.201 | 0.465 | 1.910 | NA |
| D2 | 0.496 | -0.017 | 0.256 | 0.322 | 1.424 |
| D3 | 0.514 | 0.183 | -0.279 | 0.488 | 1.665 |
| E1 | 0.123 | -1.401 | -0.022 | 0.339 | 1.577 |
| E2 | 0.437 | -0.534 | -0.314 | 1.067 | 1.526 |
| E3 | 0.224 | -0.651 | -0.154 | 0.324 | 1.376 |

Of the 17 items, there are 15 items that have 4 thresholds and 2 items that have 3 thresholds. Generally, threshold $1<$ threshold $2<$ threshold $3<$ threshold 4 , because in response to "strongly agree" it should have a higher tendency of approval than responding to "agree". There are 13 items that have a threshold that is getting bigger from one category to the next. The Characteristic Curve items of several items that have a threshold like this are presented in Figure 4.

PCM does not require the steps to complete the test items to be sequential, nor for them to have the same difficulty (De Ayala, 1993). This resulted in the fact that


Figure 4. Items with threshold $1<$ threshold $2<$ threshold $3<$ threshold 4
the threshold for PCM scoring from one category to the next is not always greater. There are 4 items, namely B1, B3, C2, and D3 which have a threshold $2<$ threshold $1<$ threshold $3<$ threshold 4 , as presented in Figure 5.

The level of difficulty in PCM is the level of difficulty answering the upper-level category for each item and is presented in Table 4.


Figure 5. Items with threshold $2<$ threshold $1<$ threshold $3<$ threshold 4

Table 4. Difficulty level for each category in each item

| Itm.Ctgr | Diff. | Itm.Ctgr | Diff. | Itm.Ctgr | Diff. | Itm.Ctgr | Diff. | Itm.Ctgr | Diff. |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| A1.c2 | -0.694 | B1.c3 | 0.122 | B4.c4 | -0.040 | C4.c2 | -1.173 | D3.c4 | 2.057 |
| A1.c3 | -0.007 | B1.c4 | 1.793 | C1.c1 | -0.707 | C4.c3 | -0.450 | E1.c1 | -1.401 |
| A1.c4 | 1.302 | B2.c1 | -1.193 | C1.c2 | -1.040 | C4.c4 | 0.919 | E1.c2 | -1.423 |
| A2.c1 | -0.604 | B2.c2 | -0.971 | C1.c3 | -0.233 | D1.c1 | 0.201 | E1.c3 | -1.083 |
| A2.c2 | -0.860 | B2.c3 | -0.463 | C1.c4 | 1.147 | D1.c2 | 0.666 | E1.c4 | 0.493 |
| A2.c3 | -0.113 | B2.c4 | 1.218 | C2.c1 | -0.417 | D1.c3 | 2.576 | E2.c1 | -0.534 |
| A2.c4 | 1.408 | B3.c1 | 0.047 | C2.c2 | -0.898 | D2.c1 | -0.017 | E2.c2 | -0.848 |
| A3.c1 | -0.691 | B3.c2 | -0.654 | C2.c3 | -0.299 | D2.c2 | 0.239 | E2.c3 | 0.220 |
| A3.c2 | -0.719 | B3.c3 | 0.302 | C2.c4 | 1.602 | D2.c3 | 0.561 | E2.c4 | 1.746 |
| A3.c3 | -0.241 | B3.c4 | 1.779 | C3.c1 | 0.044 | D2.c4 | 1.985 | E3.c1 | -0.651 |
| A3.c4 | 1.502 | B4.c1 | -1.850 | C3.c2 | 0.596 | D3.c1 | 0.183 | E3.c2 | -0.805 |
| B1.c1 | -0.054 | B4.c2 | -2.519 | C3.c3 | 1.904 | D3.c2 | -0.096 | E3.c3 | -0.480 |
| B1.c2 | -0.771 | B4.c3 | -1.664 | C4.c1 | -0.877 | D3.c3 | 0.392 | E3.c4 | 0.896 |

The distribution of difficulty levels is presented in Figure 6.
The characteristics of students is presented in Figure 7.


Figure 6. The difficulty level of categories for each item


Figure 7. Characteristics of students

## Discussion

This research aims to get the right instruments to measure the attitudes of students of the Mathematics Education Study Program towards mathematics. The development of the instrument was carried out following 12 steps from developing an overall plan to reporting the results (Downing, 2006: 4-23). The purpose of a test is important in determining what content is appropriate, and what is not, what is suitable for the test, in choosing the format of the test item and how to respond, and in planning data collection efforts to evaluate the validity and interpretation of test scores, and the quality of other technical tests (Linn, 2006: 28).

Good instruments must be made in such a way that they are easy to use and the results are accurate for the purpose of the assessment. Validity refers to how far the test score provides accurate information for making decisions based on that score (Messick, 1995: 13; Amy, 1999: 3). A statement of validity cannot be made about the instrument, but must refer to conclusions made when using scores from the scale (Finch \& French, 2019:172). Therefore, the validation process involves gathering evidence to show the scientific basis for interpreting the score as planned. Validity is support for the interpretation of test scores against the intended use of tests based on evidence and theory (Mardapi, 2017:32).

Validity is defined as a framework of three interrelated aspects, namely criteria validity, content validity, and construct validity (Croker \& Algina, 1986: 217; Amy, 1999:3-5). Criteria validity means how far the score from the assessment or test is associated with certain criteria. Content validity means that the assessment should include a representative content domain. Construct validity means how far a test score correlates with the theoretical characteristics of the measured attribute. The third proof of validity is carried out in this study, namely through the validator assessment analyzed with Aiken's coefficient, Exploratory Factor Analysis (EFA), and Confirmatory Factor Analysis (CFA). Proof of validity is not compartmentalized but is used together with other measures to build arguments that show to what extent valid conclusions can be made with scores derived from these instruments (Finch \& French, 2019: 172).

The validity of the contents is determined using the agreement of experts in the field of study. The results of validation show all items are valid because of the coefficient value of Aiken's $\mathrm{V} \geq 0.75$ with 8 validators, 4 rating categories, and a significance level of 0.05 (Aiken, 1985). Although this type of validity assessment has some interesting theoretical properties, this validity will have problems with subjectivity (Kane, 2006). Construct validation rests on the foundation of theoretical expectations, and the extent to which the scale fits these expectations. In other words, the construct validity addresses the extent to which the instrument behaves according to how the theory suggests it should. Proof of construct validity can be demonstrated by analyzing results of empirical measurement, namely by Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA) (Retnawati, 2017: 20). The EFA results show that this instrument explains 63.38\% of students' attitudes towards mathematics with 5 factors. The rotation results grouped the items into 5 factors by the theory that attitude consists of 5 aspects, namely: intrinsic motivation, enjoyment, anxiety, self-confidence, and value.

The CFA results show that each path is significant from both the latent variable (exogenous variable to endogenous variable) and the observed variable, but the
$p$-value $=0.008<0.05$. This shows that the model compatibility test is not significant, so it needed to be modified. Modification was done by correlating the error covariance between the 2 observed variables, namely B3-D3, and C3-D2. This meant a review of the attitude scale indicators needed to be done. Indicator B3 is "a challenge to solve a new problem", while indicator D3 is "can be a good problem solver". The two indicators measure related things, namely attitudes towards problem-solving. To be a good problem solver, students must have the challenge to solve new problems. The C3 indicator is "confusion", while the D2 indicator is "can learn mathematics easily". These two indicators also measure related things. Students who can learn mathematics easily will certainly not experience confusion. This relationship can also be reversed, students who do not experience confusion will be able to learn mathematics easily.

A Partial Credit Model (PCM) is very suitable for analyzing multipoint scale personality response scales (Masters \& Wright, 1996, Embretson \& Reise, 2000). The results of the analysis showed PCM fit, meaning that the items on the attitude scale followed the 1-PL (Parameter Logistic) model by taking into account the level of difficulty and assuming constant power differences (Masters, 1982). Generally, higher category scores indicate higher ability than lower category scores. PCM is suitable for items that are scored in a tiered category, but the difficulty index in each step does not need to be ordered. The response items in this study can be correct at certain steps but can be wrong at other steps.

The threshold shows the meeting point of two category probability lines in one item. The individual's probability of responding to category $x$ at this stage is the difference between the level trait $(\theta)$ and the threshold $\left(\delta_{i j}\right)$. In other words, the category intersection parameter can be considered as the level of difficulty of the stages relating to the transition from one category to the next category, and there is a difficulty step $m i$ (intersection) for items with $m i+1$ answer categories (Embretson \& Reise, 2000). The value $\delta_{i j}$ does not always have to be sequential in item $i$ because it is a relative magnitude of two adjacent probabilities (De Ayala, 1993; Muraki, 1992). The threshold can also be interpreted as a point on a latent nature scale, where for two consecutive categories the response curves intersect. The threshold is a point where two categories have the same probability to be chosen because of the associated level of a trait (Linacre, 2006).

## Conclusions

Based on the theory, a scale to measure students' attitudes toward mathematics was compiled consisting of 5 aspects, namely: intrinsic motivation, enjoyment, anxiety, self-confidence, and value. The results of factor analysis show that the attitude scale produces a model that fits the requirements for a unidimensional measure. The results of the analysis of grain characteristics identified 65 categories of 17 items with a level of difficulty $(d)$ at intervals of $-2.52 \leq d \leq 2.58$, and most were located at intervals of $-1 \leq d<0$ of $43.08 \%$. Of 227 students, there were 19 ( $8.37 \%$ ) people who did not fit the construct of this instrument. Student attitude toward mathematics is described as student ability $(\theta)$ located at intervals of -0.67 $\leq \theta \leq 2.36$, and most student's ability is at interval $1 \leq \theta<1.5$ (47.14\%). This instrument can be used in class assessments or large-scale assessments. The results of the analysis of item characteristics can show the level of difficulty in each item of each category in detail, so the instrument can be used to measure student attitudes based on their ability level.

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